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Numerical investigation of a physically nonlinear problem of the longitudinal bending of the sandwich plate with a transversal-soft core

Badriev I., Makarov M., Paimushin V.
Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

Abstract

© PNRPU. In this paper, a numerical investigation of a physically nonlinear problem of the longitudinal bending of an infinitely long sandwich plate with a transversal-soft core is carried out. We assume that in the right face section the edges of the carrier layers are clamped and there is no adhesive joint of the core with the support element, in the left face section the edges of the carrier layers of the plates are hinge supported on a completely rigid in the transverse direction diaphragms, glued with the end section of the core. The problem is considered in the one-dimensional geometrically nonlinear statement. It is assumed that the relationship between the tangential stress and strain shear corresponds to the ideal elasticplastic models, i.e., the tangential stress modules in the core do not exceed a certain limiting value. This condition means the prevention of the structural failure and corresponds to an account of the physical nonlinearity in the core material by the ideal elastic-plastic model. The generalized statement is formulated as a problem of finding a saddle point of the Lagrange generalized functional. Lagrange functional properties are investigated. Its convexity, lower semicontinuity and coercivity on the basic variables (displacements of the points of the middle surface of the carrier layers), the concavity, upper semicontinuity and anticoercivity on the Lagrange multipliers (tangential stresses in the core) are established. It made it possible to use the general theory of the existence of saddle points to prove the existence and uniqueness theorem. To solve the problem the two-layer iterative Uzawa method is proposed, each step of which is reduced to the solving of the linear elasticity problem and finding the projection onto the convex closed set. We have established the convergence of the method. By using the software package developed in Matlab environment, the numerical experiments for a model problem have been carried out. The analysis of the results is made. The numerical results correspond to the physical picture.

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Keywords

Convergence theorem, Generalized statement, Iterative method, Mathematical simulation, Numerical experiment, Physically nonlinear problem, Sandwich plate, Solvability theorem, Transversely soft core

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